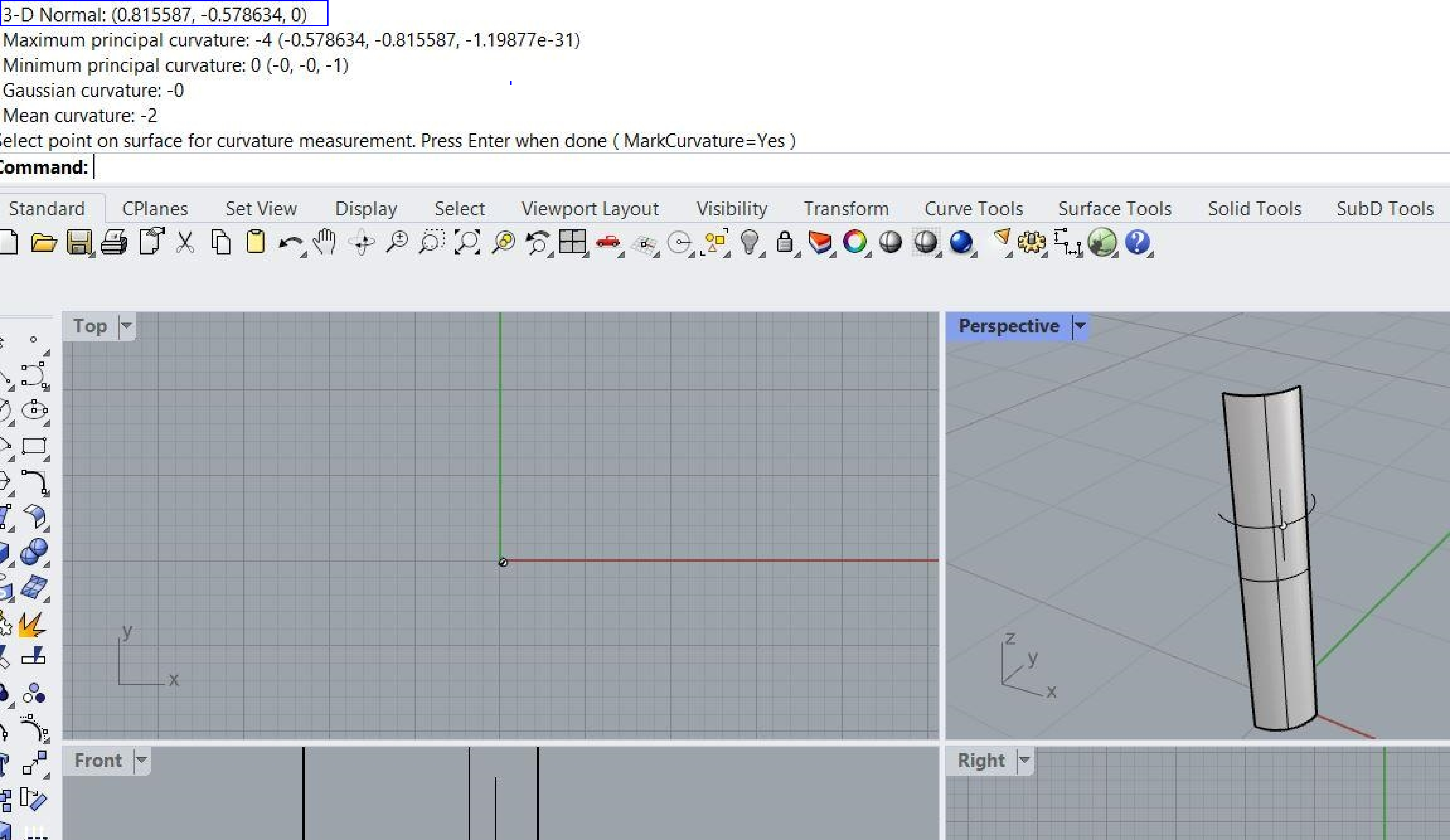
```
unit normal vector is[0] is 0.834013
unit normal vector is[1] is -0.551745
unit normal vector is[2] is 0
```

```
first principal curvature is 0
Second principal curvature is -5.32221
```


3-D Normal: (0.815587, -0.578634, 0)
Maximum principal curvature: -4 (-0.578634, -0.815587, -1.19877e-31)
Minimum principal curvature: 0 (-0, -0, -1)
Gaussian curvature: -0
Mean curvature: -2
Select point on surface for curvature measurement. Press Enter when done (MarkCurvature=Yes)
Command:
Standard
CPlanes
Set View
Display
Select
Viewport Layout
Visibility
Transform
Curve Tools
Surface Tools
Solid Tools
SubD Tools
Top
y
x
Perspective
z
y
x
Front
Right

Gerald Farin - Curves and Surfaces for CAGD_ A Practical Guide, Fifth Edition (2001)

$$\frac{\partial}{\partial u}\mathbf{b}^{m,n}(u,v) = m\sum_{j=0}^{n}\sum_{i=0}^{m-1}\Delta^{1,0}\mathbf{b}_{i,j}B_i^{m-1}(u)B_j^n(v).$$

Here we have generalized the standard difference operator in the obvious way: the superscript $(1,0)$ means that differencing is performed only on the first subscript: $\Delta^{1,0}\mathbf{b}_{i,j} = \mathbf{b}_{i+1,j} - \mathbf{b}_{i,j}$. If we take v-partials, we employ a difference operator that acts only on the second subscripts: $\Delta^{0,1}\mathbf{b}_{i,j} = \mathbf{b}_{i,j+1} - \mathbf{b}_{i,j}$. We then obtain

$$\frac{\partial}{\partial v}\mathbf{b}^{m,n}(u,v) = n\sum_{i=0}^{m}\sum_{j=0}^{n-1}\Delta^{0,1}\mathbf{b}_{i,j}B_j^{n-1}(v)B_i^m(u).$$

We can write down formulas for higher-order partials:

$$\frac{\partial^r}{\partial u^r}\mathbf{b}^{m,n}(u,v) = \frac{m!}{(m-r)!}\sum_{j=0}^{n}\sum_{i=0}^{m-r}\Delta^{r,0}\mathbf{b}_{i,j}B_i^{m-r}(u)B_j^n(v) \tag{14.10}$$

and

$$\frac{\partial^s}{\partial v^s}\mathbf{b}^{m,n}(u,v) = \frac{n!}{(n-s)!}\sum_{i=0}^{m}\sum_{j=0}^{n-s}\Delta^{0,s}\mathbf{b}_{i,j}B_j^{n-s}(v)B_i^m(u). \tag{14.11}$$

Here, the difference operators are defined by

$$\Delta^{r,0}\mathbf{b}_{i,j} = \Delta^{r-1,0}\mathbf{b}_{i+1,j} - \Delta^{r-1,0}\mathbf{b}_{i,j}$$

and

$$\Delta^{0,s}\mathbf{b}_{i,j} = \Delta^{0,s-1}\mathbf{b}_{i,j+1} - \Delta^{0,s-1}\mathbf{b}_{i,j}.$$

It is not hard now to write down the most general case, namely, *mixed partials* of arbitrary order:

$$\frac{\partial^{r+s}}{\partial u^r \partial v^s}\mathbf{b}^{m,n}(u,v)$$
$$= \frac{m!n!}{(m-r)!(n-s)!}\sum_{i=0}^{m-r}\sum_{j=0}^{n-s}\Delta^{r,s}\mathbf{b}_{i,j}B_i^{m-r}(u)B_j^{n-s}(v). \tag{14.12}$$